Suggested Solutions to:
Resit Exam, Spring 2019
Industrial Organization
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Question 1: Selling a durable good

(a) Solve for the equilibrium value of $\widehat{r}$ and show that the solution you have found is indeed an equilibrium. You may assume that the second-order conditions are satisfied.

As stated in the question, we look for an equilibrium that is characterized by a cutoff value $\widehat{r}$, such that a consumer buys in the first period if and only if $r \geq \widehat{r}$. We can solve for such an equilibrium by considering all the stages of the model where an economic agent (the firm or a consumer) makes a choice, and ensure that these choices are made optimally (given that the agent correctly anticipates decisions made later in the game).

- *Second period.* The firm faces the demand $q_2 = \widehat{r} - p_2$, which yields the profits $\pi_2 = (\widehat{r} - p_2)(p_2 - c)$. These are maximized at

$$p_2 = \frac{\widehat{r} + c}{2}. \tag{1}$$

- *First period, second stage.* A consumer weakly prefers to purchase if and only if

$$r(1+\delta) - p_1 \geq \delta(r - p_2).$$

By using (1) and simplifying, we can equivalently write this inequality as $2r \geq 2p_1 - \delta\widehat{r} - \delta c$. The cutoff value $\widehat{r}$ must thus be characterized by $2\widehat{r} = 2p_1 - \delta\widehat{r} - \delta c$, or:

$$\widehat{r} = \frac{2p_1 - \delta c}{2+\delta} \Leftrightarrow p_1 = \frac{(2+\delta)\widehat{r} + \delta c}{2}. \tag{2}$$

- *First period, first stage.* The firm chooses $\widehat{r}$ so as to maximize its overall profits[1]

$$\begin{aligned}
\Pi &= \pi_1 + \delta\pi_2 \\
&= (1-\widehat{r})(p_1 - c) + \delta(\widehat{r} - p_2)(p_2 - c) \\
&= (1-\widehat{r})\frac{(2+\delta)\widehat{r} - (2-\delta)c}{2} + \delta\frac{(\widehat{r}-c)^2}{4}.
\end{aligned}$$

The first-order condition is

$$\begin{aligned}
\frac{\partial\Pi}{\partial\widehat{r}} &= -\frac{(2+\delta)\widehat{r} - (2-\delta)c}{2} \\
&\quad + (1-\widehat{r})\frac{2+\delta}{2} + \delta\frac{\widehat{r}-c}{2} = 0 \\
&\Leftrightarrow \widehat{r}^S = \frac{2+\delta+2(1-\delta)c}{2(2+\delta)-\delta}.
\end{aligned} \tag{3}$$

Our analysis is valid only for $\widehat{r} \in (0,1)$. By working through some straightforward algebra, one can verify that we have both $\widehat{r} > 0$ and $\widehat{r} < 1$. Thus, we can conclude that (3) indeed is an equilibrium value of $\widehat{r}$.

(b) Denote total surplus (i.e., the sum of firm profit and consumer surplus) for the market in period t by W_t, for $t = 1, 2$. Write up expressions for W_1 and W_2, as functions of $\widehat{r}$, p_1, and p_2 (i.e., do not plug in the equilibrium values of this cutoff value and these prices).

- You are encouraged to attempt this question also if you have failed to answer part (a).

In period 1, consumers with $r \in [\widehat{r}, 1]$ purchase and consume the good, and the production cost is c. Therefore, total surplus in period 1 is given by

$$W_1 = \int_{\widehat{r}}^{1}(r-c)dr.$$

[1] Letting the firm choose $\widehat{r}$ instead of p_1 is more convenient (it simplifies the algebra) and it does not change the results.

Alternatively, total surplus in period 1 can be written as the sum of first-period profits and consumer surplus or $W_1 = \Pi_1 + CS_1$, where $\Pi_1 = (1-\widehat{r})(p_1 - c)$ and

$$CS_1 = \int_{\widehat{r}}^{1} (r - p_1) dr.$$

In period 2, consumers with $r \in [\widehat{r}, 1]$ again consume the good, since it is still in their possession and it is durable; however, there is no new production cost associated with this consumption. In addition, consumers with $r \in [p_2, \widehat{r}]$ consume the good in period 2, and the production cost associated with this consumption is c. Therefore, total surplus in period 2 is given by

$$W_2 = \int_{p_2}^{\widehat{r}} (r - c) dr + \int_{\widehat{r}}^{1} r dr.$$

Alternatively, total surplus in period 2 can be written as the sum of second-period profits and consumer surplus or $W_2 = \Pi_2 + CS_2$, where $\Pi_2 = (p_2 - c)(\widehat{r} - p_2)$ and

$$CS_2 = \int_{p_2}^{\widehat{r}} (r - p_2) dr + \int_{\widehat{r}}^{1} r dr.$$

(c) What were these two effects? Explain briefly the logic of the effects and why and how they improved welfare.

- You should not show any mathematics when answering this question (and you will not get any credit if you nevertheless do that).
- You are encouraged to attempt this question also if you have failed to answer parts (b) and (c).

From the lecture slides:

- Why did we get the result that BBPD is good for consumers as a group (using only the Pareto criterion)?
- Two mechanisms that tend to create surplus for the consumers:
  1. Price discrimination makes it profitable for the firm to **trade with consumers with a low valuation**. The firm can charge a separate, lower, price to those consumers, without having to sell to the high-valuation consumers at the same price.
  2. The firm loses market power due to the **"durable-good logic"**: The consumers are patient and forward-looking. They know that if they buy early, the second-period price will increase for them. Therefore they buy in period 1 only if the price is sufficiently low. This forces the firm to indeed lower the first-period price.

Question 2: Collusion in a Cournot oligopoly with a fixed production cost

To the external examiner: This question is identical to (parts of) a question in a problem set that the students discussed in an exercise class.

Part (a)

We must investigate under what conditions each one of the firms does not have an incentive to deviate from the strategy. In qualitative terms, there are three different situations we need to consider: (i) on the equilibrium path, the firm that is supposed to choose $q_{i,t} = 0$ must not have an incentive to deviate; (ii) on the equilibrium path, the firm that is supposed to choose $q_{i,t} = 6$ must not have an incentive to deviate; (iii) off the equilibrium path, neither firm must have an incentive to deviate from $q_{i,t} = 4$.

In situation (iii) it is clear that no firm would have an incentive to deviate, simply because $(q_{1,t}, q_{2,t}) = (4, 4)$ is a Nash equilibrium of the one-shot game. If expecting the other firm (say firm j) to choose $q_{j,t} = 4$, then the action that maximizes the current-period profits is indeed $q_{i,t} = 4$ (and the rival's actions in future periods will not change if deviating from $q_{i,t} = 4$).

Now consider situation (i): the incentives to deviate for a firm that is supposed to produce nothing. The present-discounted stream of profits for this firm, at the point when it is supposed to choose $q_{i,t} = 0$, equals

$$\begin{aligned} V^{eq} &= 0 + \delta\pi^m + 0 + \delta^3\pi^m + 0 + \delta^5\pi^m + \cdots \\ &= \delta\pi^m \left(1 + \delta^2 + \delta^4 + \delta^6 + \cdots\right) = \frac{\delta\pi^m}{1-\delta^2}, \end{aligned}$$

where π^m denotes the single-period monopoly profits:

$$\pi^m = (12 - q_1^*)\, q_1^* - k = (12 - 6)\, 6 - 8 = 28.$$

If deviating, the present-discounted stream of profits for this firm, at the point when it is supposed to choose $q_{i,t}=0$, equals

$$\begin{aligned} V^{dev} &= \pi^d + \delta\pi^n + \delta^2\pi^n + \delta^3\pi^n + \cdots \\ &= \pi^d + \delta\pi^n \left(1 + \delta + \delta^2 + \delta^3 + \cdots\right) \\ &= \pi^d + \frac{\delta\pi^n}{1-\delta}, \end{aligned}$$

where π^d denotes the best possible deviation profit if the other firm produces $q_{j,t}=6$,

$$\pi^d = \left(12 - q_1^* - q_2^*\right) q_1^* - k = (12-6-3)\,3 - 8 = 1,$$

and π^n denotes a firm's profit in the symmetric Nash equilibrium of the one-shot game,

$$\pi^n = \left(12 - q_1^* - q_2^*\right) q_1^* - k = (12-4-4)\,4 - 8 = 8.$$

So there is no incentive to deviate if

$$\begin{aligned} V^{eq} \geq V^{dev} &\Leftrightarrow \frac{\delta\pi^m}{1-\delta^2} \geq \pi^d + \frac{\delta\pi^n}{1-\delta} \\ &\Leftrightarrow \delta\pi^m \geq \left(1-\delta^2\right)\pi^d + (1+\delta)\,\delta\pi^n. \end{aligned}$$

Using the above values for π^m, π^d and π^n, this condition simplifies to

$$28\delta \geq \left(1-\delta^2\right) + 8\,(1+\delta)\,\delta \Leftrightarrow f(\delta) \geq 0,$$

where

$$f(\delta) \equiv 28\delta - \left(1-\delta^2\right) - 8\,(1+\delta)\,\delta.$$

Note that we have $-1 = f(0) < 0 < f(1) = 12$ and

$$\begin{aligned} f'(\delta) &\equiv 28 + 2\delta - 8 - 16\delta \\ &= 14\,(1-\delta) + 6 > 0 \end{aligned}$$

for all $\delta < 1$. This means that there is a unique cut-off value $\delta_0 \in (0,1)$, defined by $f(\delta_0) = 0$, such that the firm that is supposed to produce nothing has no incentive deviate if, and only if, $\delta \geq \delta_0$.

What about situation (ii)? That is, what about the incentives to deviate for a firm that is supposed to produce $q_{i,t}=6$? It may look as if such a firm should, if expecting the rival to choose $q_{j,t}=0$, never have an incentive to deviate, because the firm would in the current period earn the monopoly profit, which cannot be made larger. However, this firm can, by deviating, improve on its profits in the *following* period (as well as all the future periods in which it is supposed to produce zero). We therefore need to investigate this case too. The firm can deviate in a way that lowers its current period profits with some arbitrarily small amount, by choosing a quantity that is slightly lower or slightly higher than $q_{i,t}=6$. If doing that, the firm's profits would equal $\pi^d = 28-\varepsilon$, where ε is some positive number that can be made arbitrarily small. This action would also trigger the punishment phase, which means that the firm would earn the profit $\pi^n = 8$ in all the subsequent periods. Overall, the firm's present-discounted stream of profits if deviating in that way equals

$$V^{dev} = \pi^d + \frac{\delta\pi^n}{1-\delta} = 28 - \varepsilon + \frac{8\delta}{1-\delta}.$$

The firm's present-discounted stream of profits if not deviating equals

$$\begin{aligned} V^{eq} &= \pi^m + 0 + \delta^2\pi^m + 0 + \delta^4\pi^m + 0 + \cdots \\ &= \pi^m\left(1 + \delta^2 + \delta^4 + \delta^6 + \cdots\right) = \frac{\pi^m}{1-\delta^2} \\ &= \frac{28}{1-\delta^2}. \end{aligned}$$

So there is no incentive to deviate if

$$V^{eq} \geq V^{dev} \Leftrightarrow \frac{28}{1-\delta^2} \geq 28 - \varepsilon + \frac{8\delta}{1-\delta},$$

which holds for all $\varepsilon > 0$ if, and only if,

$$\frac{28}{1-\delta^2} \geq 28 + \frac{8\delta}{1-\delta}.$$

Simplifying this inequality yields

$$\begin{aligned} 28 \geq 28\left(1-\delta^2\right) + 8\delta\,(1+\delta) &\Leftrightarrow 28\delta \geq 8\,(1+\delta) \\ &\Leftrightarrow \delta \geq \frac{8}{20} = 0.4. \end{aligned}$$

That is, the firm that is supposed to produce $q_{i,t} = 6$ does not have an incentive to deviate if and only if $\delta \geq 0.4$. Moreover, this condition is more stringent than the one required in situation (i) above: $\delta_0 < 0.4$. (This follows because $f'(\delta) > 0$ and $f(0.4) > 0$.)

Overall we can conclude that the specified strategies constitute a subgame perfect Nash equilibrium if and only if $\delta \geq 0.4$.

Part (b)

From the lecture slides:

- The result that cooperation is possible for large enough values of δ is a special case of a more general result called the **Folk Theorem**.

- The **Folk Theorem**: In an infinitely repeated game with observable actions and in which the players are sufficiently patient:
  - Everything (that is feasible and individually rational) is an equilibrium.
- The Folk Theorem is, in a way, a problem for the theory:
  - What is the theory's prediction? If we can explain everything, then we cannot explain anything!
- The (pragmatic) approach taken by IO economists:
  - Assume the players can coordinate their behavior on some **"focal" equilibrium**.
  - For example, in a symmetric game, the players **coordinate on a symmetric equilibrium, and this equilibrium is Pareto efficient** from the point of view of these players (e.g., the firms).